

Non-Static Cosmological Model with Cylindrical Symmetry in Presence of Electromagnetic Field

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ABSTRACT

The present paper provides the electromagnetic field is non-null and consists of an electric and/or magnetic field both of which are in the direction of same space axis. In the magnetohydrodynamics case metric functions are restricted not only by field equations and energy conditions, they are also restricted by the requirements that the conductivity by positive for realistic model.

1. INTRODUCTION

Various relativists have focused their mind to the study of electromagnetro hydrodynamics in recent years. As a matter of fact, it is well known that galaxies and interstellar spaces exhibit the presence of strong magnetic fields [28] which impart a sort of viscous effect in the fluid flow [3]. The magnetic field assumes an important role for the universe. The behavior of the magnetic field of a star was investigated by Cowling [4] and Wrubel [24]. The important results obtained make possible the interpretation of magnetohydrodynamical processes in stars. When motion of stellar matter caused by the electromagnetic forces are taken into account, new properties may be revealed and the non-stability of the magnetohydrodynamical processes in stars may be studied. A cosmological model in the presence of magnetic field has been studied by Zeldovich [27] and later by Thorne [20]. Skikin [18] also constructed a uniform axially symmetric solution (model) of Einstein-Maxwell equations in the case of propagation by an ideal fluid in the presence of magnetic field directed along the axis of symmetry. Magnetic field in stellar bodies was also discussed by Monaghan [13]. Gravitationally collapse of a magnetic star was studied by Greenburg [7]. Jacobs [9, 10] has studied the behavior of the general Bianchi-type I cosmological model in the presence of a spatially homogeneous magnetic field. This problem has been studied again by De [5] with a different approach. This work has been further extended by Tupper [22] to include Einstein-Maxwell fields in which the electric field is non-zero. He has also interpreted certain type VI cosmologies with electromagnetic field (Tupper [23]).

Bali and Jain [1] have constructed magneto static models filled with dust and disordered radiation in which the distribution is that of a perfect fluid and have found some physical and geometrical aspects of the model. Roy and Prakash [17] taking the cylindrically symmetric metric of Marder [12] have constructed a spatially homogeneous cosmological model in the presence of an incident magnetic field which is also anisotropic and non-degenerate Petrov type.

Singh and Yadav [19] constructed a spatially homogeneous cosmological model assuming the energy momentum tensor to be that of a perfect fluid with an electromagnetic field. Yadav and Purushottam [26] have also considered a non-static cosmological model with an electromagnetic field with different assumptions. Some other workers in this line are Yadav and Singh [25], Garecki [6 (a)], Paul [29a, b], Bergaman [2], Rendal [15], Bali and Yadav [31] and Pradhan and Vishwakarma [30], Roy and Prakash [17], Bhattacharjee & Baruch [1(a)].

Here in this chapter, considering the cylindrically symmetric metric of Marder, we have also constructed a non static spatially homogeneous Petrov type. I cosmological model assuming the energy momentum tensor to be that of a perfect fluid with an electromagnetic field and the four current is either zero or space like. Our model is more general than one due to Singh and Yadav [19] and yadav and Purushottam [26]. The requirement that the conductivity be positive imposes an additional condition on the metric functions. Various physical and geometrical properties e.g. pressure, density, rotation scalar of expansion and components of shear tensor have been found. We have also discussed Doppler effect and a Newtonian analogue of force in the model. We have shown that when cosmological term $\Lambda = 0$ then in the absence of electromagnetic field pressure and density becomes equal and conversely if pressure and density are equal there is no electromagnetic field.

2. THE FIELD EQUATIONS AND THEIR SOLUTIONS

We consider the most general cylindrically symmetric space time in the form given by Marder [12].

$$(2.1) \quad ds^2 = A^2 (dt^2 - dx^2) - b^2 dy^2 - C^2 dz^2$$

where the metric potentials A, B, C are functions of time t alone. This ensures that the model is spatially homogeneous. The non-vanishing components of Ricci tensor R_j^i and Weyl conformal curvature tensor for this metric have been given in the appendix of this chapter.

The distribution consists of a perfect fluid and an electromagnetic field. The energy momentum tensor of the composite field is assumed to be the sum of the corresponding energy momentum tensors. Thus

$$(2.2) \quad R_{ij} = \frac{1}{2} g_{ij} R + \wedge g_{ij} = -K [(\rho + p)u_i u_j - p g_{ij} + E_{ij}],$$

$$(2.3) \quad g_{ij} u^i u^j = 1$$

$$(2.4) \quad E_{ij} = g^{kl} F_{ik} F_{jl} - \frac{1}{4} g_{ij} F_{mn} F^{mn}$$

$$(2.5) \quad F_{[ij;k]} = 0$$

$$(2.6) \quad F^{ij}; j = j^i$$

where E_{ij} is the electromagnetic energy momentum tensor, F_{ij} is the electromagnetic field tensor, \wedge is cosmological constant, j^i is current four vector and ρ and p are respectively, the density and pressure of the distribution. The co-ordinates are chosen to be co-moving so that

$$(2.7) \quad u^1 = u^2 = u^3 = 0 \text{ and } u^4 = \frac{1}{A}$$

We label the co-ordinates $(x, y, z, t) = (x^1, x^2, x^3, x^4)$.

The off-diagonal components of (2.2) are

$$(2.8a) \quad F_{12} F_{24} B^{-2} + F_{13} F_{34} C^{-2} = 0$$

$$(2.8b) \quad F_{12} F_{14} A^{-2} - F_{23} F_{34} C^{-2} = 0$$

$$(2.8c) \quad F_{13} F_{13} A^{-2} + F_{23} F_{24} B^{-2} = 0$$

$$(2.8d) \quad F_{14} F_{24} A^{-2} - F_{13} F_{23} C^{-2} = 0$$

$$(2.8e) \quad F_{14} F_{34} A^{-2} + F_{12} F_{23} B^{-2} = 0$$

$$(2.8f) \quad F_{24} F_{34} - F_{12} F_{13} = 0$$

which lead to three possible cases :

(i) $F_{24} = F_{34} = F_{12} = F_{13} = 0$ at least one of F_{14}, F_{23} non-zero i.e., when the field F_{ij} is in X-direction only.

(ii) $F_{14} = F_{34} = F_{12} = F_{23} = 0$ at least one of F_{24}, F_{13} non-zero i.e. when the field is in Y-direction only.

(iii) $F_{14} = F_{24} = F_{13} = F_{23} = 0$ at least one of F_{34}, F_{12} non-zero i.e. when the field is in Z-direction only.

Hence the electromagnetic field is non-null and consists of an electric and/or magnetic field both of which are in the direction of same space axis. Without loss of generality, we may consider only case (i) in which the electric and magnetic field are in the X-direction. We write.

$$(2.9) \quad F_{14}^2 A^{-4} + F_{23}^2 B^{-2} C^{-2} = L^2$$

The diagonal components of the equation (2.2) may be written as

$$(2.10) \quad \frac{2}{A^2} \left[\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{A_4 C_4}{AC} - \frac{A_4 B_4}{AB} - \frac{A_4^2}{A^2} \right] - 2 \wedge = -K [L^2 + (\rho + 3p)]$$

$$(2.11) \quad \frac{2}{A^2} \left[\frac{A_{44}}{A} + \frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} - \frac{A_4^2}{A^2} \right] + 2 \wedge = K [-L^2 + (\rho - p)]$$

$$(2.12) \quad -\frac{2}{A^2} \left[\frac{B_{44}}{B} + \frac{B_4 C_4}{BC} \right] + 2 \wedge = -K [L^2 + (\rho - p)]$$

$$(2.13) \quad -\frac{2}{A^2} \left[\frac{C_{44}}{C} + \frac{B_4 C_4}{BC} \right] + 2 \wedge = -K [L^2 + (\rho + p)]$$

where the suffix 4 indicates the ordinary differentiation with respect to time t after the symbols A, B, C. From these equations it is clear that L^2, \square, p are each functions of time t alone.

From equation (2.5) and (2.9) it follows that F_{23} is a constant and F_{14} is a function of time t only i.e.

$$(2.14) \quad F_{23} = k, F_{14} = \pm A^2 (L^2 - k^2 B^{-2} C^{-2})^{1/2}$$

where k is a constant. In the case when $F_{14} = 0$ which implies $j^i = 0$, we get the model due to Roy and Prakash [17]. Here we assume that $F_{14} \neq 0$ and find the only non-vanishing components of j^i to be

$$(2.15) \quad j^i = \pm \frac{1}{A^2 BC} \frac{\partial}{\partial t} \left[BC(L^2 - k^2 B^{-2} C^{-2})^{1/2} \right]$$

Equation (2.15) shows that J^i is space-like, unless $L^2 = IB^{-2} C^{-2}$

Where 1 is a constant in which case $j^i = 0$

The four-current j^i is in general the sum of the convection current and conduction current (Licknerowics) [11] and Greenberg [7].

$$(2.16) \quad j^i = \epsilon_0 u^i + \zeta u_j F^{ij}$$

where ϵ_0 is the rest charge density and ζ is the conductivity. In the case considered here we have $\epsilon_0 = 0$ i.e. magnetohydrodynamics. From equation (2.14), (2.15) and (2.16) we find the conductivity is given by

$$(2.17) \quad \zeta = \frac{1}{A} I_4 I^{-1}$$

where $I = BC(L^2 - k^2 B^{-2} C^{-2})^{1/2}$

The requirement of positive conductivity in (2.17) puts further restrictions on A, B, C . Hence in the magnetohydrodynamics case metric functions are restricted not only by field equations and energy conditions (Hawking and Penrose [8]), they are also restricted by the requirement that the conductivity by positive for realistic model.

The equations (2.10) – (2.13) are four equations in six unknown A, B, C, \square, p and L . In order to determine them, two more conditions have to be imposed on them. For this we assume that the space time is of degenerate Petrov type – 1, the degeneracy being in y and z directions. This requires that $C_{12}^{12} = C_{13}^{13}$. This condition is identically satisfied if $B = C$. However, we shall take metric potentials to be unequal. We further assume that F_{14} is such that

$$L^2 = H^2 / (BC)^6$$

where H is a constant.

From equation (2.12) and (2.13) we have

$$(2.18) \quad \frac{B_{44}}{B} - \frac{C_{44}}{C} = 0$$

Equation (2.18) with condition $C_{12}^{12} = C_{13}^{13}$ gives

$$(2.19) \quad \frac{A_4}{A} \left(\frac{C_4}{C} - \frac{B_4}{B} \right) = 0$$

Since $B \neq C$, equation (2.18) gives

$$(2.20) \quad A = M \text{ (an arbitrary constant)}$$

From equation (2.11), (2.12) and (2.20) we have

$$(2.21) \quad \frac{B_{44}}{B} + \frac{B_4 C_4}{BC} = KL^2 M^2$$

Equation (2.18) on integration yields.

$$(2.22) \quad B_4 C - BC_4 = \square$$

\square being an arbitrary constant of integration. On putting $\frac{B}{C} = \lambda$ and $BC = v$ equation (2.22) goes to the form

$$(2.23) \quad \left(\frac{\lambda_4}{\lambda} \right) v = \alpha$$

and equation (2.21) turns into

$$(2.24) \quad \frac{1}{v} \left[\left(\frac{\lambda_4}{\lambda} + \frac{v_4}{v} \right) v \right]_4 = 2KL^2 M^2$$

From equation (2.23) and (2.24) we have

$$(2.25) \quad \frac{v_{44}}{v} = 2KL^2 M^2$$

which, after the use of condition $L^2 = H^2 / (BC)^6$ reduces to

$$(2.26) \quad v_{44} = 2KH^2 M^2 v^{-5}$$

Equation (2.26) on integration yields

$$(2.27) \quad [v_4]^2 = \frac{4KM^2H^2 v^8}{10} + \beta^2$$

where \square is constant of integration. From (2.23) and (2.27). We get

$$(2.28) \quad \left(\frac{d\lambda}{\lambda}\right) = \frac{\alpha}{\beta} \cdot \frac{dv}{(v^4 + D^2)^{1/2}}$$

where

$$(2.29) \quad D^2 = \frac{4KM^2H^2}{\beta^2(\mu - 2)} = \frac{4KM^2H^2}{4\beta^2}$$

Integration of (2.28) gives.

$$(2.30) \quad \lambda D_1 = e^{\frac{\alpha}{\beta}\psi(v)}$$

where D_1 is constant of integration and

$$(2.31) \quad \psi(v) = \int \frac{dv}{\sqrt{V^4 + D^2}}$$

Hence we have

$$(2.32) \quad B^2 = D_1 v e^{\frac{\alpha}{\beta}\psi(v)}$$

and

$$(2.33) \quad C^2 = \frac{v}{D_1} e^{-\frac{\alpha}{\beta}\psi(v)}$$

Therefore the metric (2.1) can be written as

$$(2.34) \quad ds^2 = A^2 \left[\frac{dv^2}{(dv/dt)^2} - dx^2 \right] - B^2 dy^2 - C^2 dz^2$$

which by the use of equations (2.20), (2.27), (2.32) and (2.33) takes the form

$$(2.35) \quad \therefore ds^2 = M^2 \left[-dx^2 + \frac{dv^2}{(\beta^2/v^4)(v^4 + D^4)} \right] - D_1 v e^{-\alpha/\beta\psi(v)} dy^2 - \frac{v}{D_1} e^{\frac{-\alpha}{\beta}\psi(v)} dz^2$$

$$(2.36) \quad V^4 + D^4 = T^4$$

reduces (2.35) to the form

$$(2.37) \quad ds^2 = M^2 \left[-dx^2 + (T^4 - D^4)^{-1/2} T^2 dT^2 \right] - D_1 (T^4 - D^4)^{1/4} \cdot e^{2hf(T)} dy^2 - \frac{(T^4 - D^4)^{1/4}}{D_1} \cdot e^{-2hf(T)} dz^2$$

where $\frac{\alpha}{2\beta} = h, \psi(v) = f(T)$

Again the use of transformation

$$x \rightarrow X, \sqrt{D_1} y \rightarrow Y, \frac{1}{\sqrt{D_1}} z \rightarrow Z$$

leads (2.37) to the form

$$(2.38) \quad ds^2 = M^2 \left[-dX^2 + (T^4 - D^4)^{-1/2} T^2 \cdot dT^2 \right] - (T^4 - D^4)^{1/4} \cdot e^{2hf(T)} dy^2$$

$$-(T^2 - D^4)^{1/4} e^{2hf(T)} dz^2$$

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